

6/15/12 PER-UNIT

IDEA: use normalized values so

1. It's easier to compare devices of different ratings
2. Reduce range of values to improve accuracy of numerical calculations

Quantity in pu = $\frac{\text{physical (SI) quantity}}{\text{base "reference" value of quantity}}$

$$\text{1 } \phi \quad I_b = \frac{S_b}{V_b} = \frac{VA_{1\phi, \text{base}}}{V_{LN, \text{base}}}$$

$$\text{where } S_b = P_b = Q_b = V_b I_b = V_{LN, b} I_b$$

$$Z_b = R_b = X_b = \frac{V_b}{I_b} = \frac{V_{LN, b}}{I_b} = \frac{V_b}{S_b/V_b} = \frac{V_b^2}{S_b} = \frac{V_{LN, b}^2}{S_b}$$

3 ϕ - A little trickier

$$S_{b3\phi} = 3 S_{b1\phi}$$

$$V_{Lb} = \sqrt{3} V_{LN, b}$$

$$P_{b3\phi} = 3 P_{b1\phi}$$

$$Q_{b3\phi} = 3 Q_{b1\phi}$$

$$I_b = \frac{S_{b3\phi}}{3 V_{Lb}}$$

$$Z_b = \frac{(V_{Lb}/\sqrt{3})^2}{S_{b3\phi}/3} = \frac{V_{Lb}^2}{S_{b3\phi}}$$

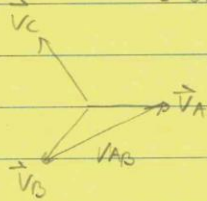
$$= \frac{S_{b3\phi}}{\sqrt{3} V_{Lb}}$$

$$\begin{aligned}
 \text{Show } \vec{V}_{AB} &= \sqrt{3} V_A = \\
 &= |\vec{V}_A - \vec{V}_B| \\
 &= |V_A \angle 0^\circ - V_A \angle -120^\circ| \\
 &= |V_A - [V_A \cos(-120^\circ) + j V_A \sin(-120^\circ)]| \\
 &\approx |V_A + \frac{1}{2} V_A + j \frac{\sqrt{3}}{2} V_A| \\
 &= \left| \frac{3}{2} + j \frac{\sqrt{3}}{2} \right| V_A
 \end{aligned}$$

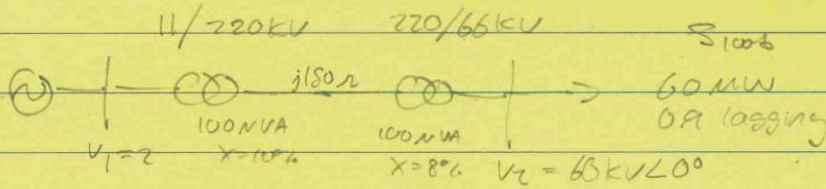
$$|\vec{V}_{AB}| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{12}{4}} = \sqrt{3}$$

$$\phi_{AB} = \cos^{-1} \frac{\text{Re } V_{AB}}{|V_{AB}|} = \cos^{-1} \left(\frac{3/2}{\sqrt{3}} \right) = \cos^{-1}(\sqrt{3})$$

$= 30^\circ$ \vec{V}_{AB} LEADS V_A by 30°



example



$$Z_{b, \text{line}} = \frac{(220 \text{ kV})^2}{100 \text{ MVA}} = 484 \text{ } \Omega \quad S_b = 100 \text{ MVA}$$

$$X_{\text{line}} = \frac{180 \Omega}{484} = 0.37 \text{ pu} \quad V_{b, \text{gen}} = 11 \text{ kV}$$

$$S_{\text{load}} = 60 + j60 \tan(\cos^{-1}(0.9)) \text{ MVA}$$

$$= 0.6 + j0.262 \text{ pu}$$

$$I_{\text{load}} = \frac{S_{\text{load}}^*}{V_{b, \text{line}}} = 0.6 - j0.262 \text{ pu} = 0.66 - j0.29$$

$$V_2 = \frac{60 \text{ kV}}{66 \text{ kV}} = 0.91 \text{ pu} \quad Z_{\text{eq}} = X_{t1} + X_{\text{line}} + X_{t2}$$

$$= j0.1 + j0.08 + 0.37$$

$$= j0.489 \text{ pu}$$

$$V_1 = V_2 + Z_{\text{eq}} I_{\text{load}}$$

$$= 0.91 + (j0.489)(0.6 - j0.262)$$

$$= 1.1 \angle -3.052^\circ$$

$$V_{1, \text{st}} = (11 \text{ kV})(1.1) = 12.1 \text{ kV}$$

Change-of-base: Idea: express as ratio wrt new re

$$Z_{\text{pu, new}} = Z_{\text{pu, old}} \left(\frac{S_{b, \text{new}}}{S_{b, \text{old}}} \right) \left(\frac{V_{b, \text{old}}}{V_{b, \text{new}}} \right)^2$$

$$(S, P, Q)_{\text{pu, new}} = (S, P, Q)_{\text{pu, old}} \left(\frac{S_{b, \text{new}}}{S_{b, \text{old}}} \right)$$

$$V_{\text{pu, new}} = V_{\text{pu, old}} \left(\frac{V_{b, \text{old}}}{V_{b, \text{new}}} \right)$$