Circuit Elements & Phasor Analysis

Circuit Elements

Resistor

$v = Ri \rightarrow p = vi = Ri^2$

Inductor

$v = L\frac{di}{dt} \rightarrow p = vi = Li\frac{di}{dt}$

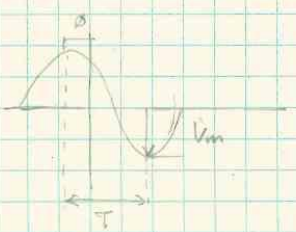
Capacitor

$i = C\frac{dv}{dt} \rightarrow p = vi = Cv\frac{dv}{dt}$

Phasor Analysis & Sinusoidal Response

Sinusoidal voltage

$v = V_m \cos(\omega t + \phi)$



$f = \frac{1}{T}$

$\omega = 2\pi f$

$\phi_{deg} = \frac{180°}{\pi}\phi_{rad}$

RMS Value

RMS current is the DC current that has the same average power dissipated through a resistor as current i(t) periodic in T

eg. $p(t) = i^2(t)R$

$P_{avg} = \frac{1}{T}\int p(t)\,dt = \frac{1}{T}\int i^2(t)R\,dt$

note that: rearranging $P_{avg} = \left[\sqrt{\frac{1}{T}\int_{T} i^2(t)\,dt}\right]^2 R$

quantity inside is RMS value so

$$P_{avg} = (I_{rms})^2 R$$

Can calculate similarly for v. Note that

~~$P_{avg} = V_{rms} I_{rms}$~~ for sinusoids in phase

For sine wave: $I_{rms} = \frac{I_m}{\sqrt{2}}$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_m \cos^2\theta \, d\theta} = \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi} \frac{1 + \cos 2\theta}{2} \, d\theta}$$

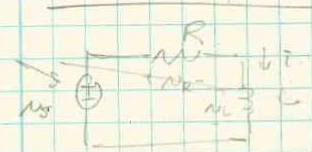
$$= \sqrt{\frac{I_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{2\cdot 2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\left(\frac{2\pi}{2} - \frac{\cancel{\sin 2\cdot 2\pi}}{2\cdot 2}\right) - \left(\frac{0}{2} - \frac{\cancel{\sin 2\cdot 0}}{2\cdot 2}\right)}$$

$$= \sqrt{\frac{I_m^2}{\cancel{2\pi}}\frac{\cancel{2\pi}}{2}} = \frac{I_m}{\sqrt{2}} \checkmark$$

draw as

Sinusoidal Response



$v_s = V_m \cos(\omega t + \phi)$

~~$v_s = v_R + v_L$~~

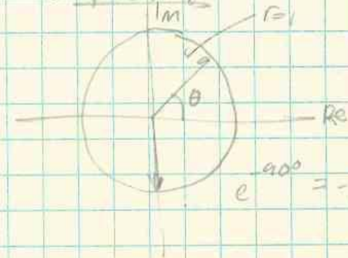
~~$V_m \cos(\omega t + \phi) = Ri + L\frac{di}{dt}$ } differential equation~~

PHASORS

Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

Graphically



$$e^{-90°} = -j$$

$$\cos\theta = Re\ e^{j\theta}$$
$$\sin\theta = Im\ e^{j\theta}$$

$$v = V_m \cos(\omega t + \phi)$$
$$= V_m\ Re\ e^{(j\omega t + \phi)}$$
$$= V_m Re\, e^{j\omega t} e^{j\phi}$$
$$= Re\{\underbrace{V_m e^{j\phi}}_{\text{phasor}} e^{j\omega t}\} = \vec{V} = V_m e^{j\phi} = P\{V_m \cos(\omega t + \phi)\}$$
$$= V_m \angle \phi$$

Circuit Elements in PHASOR domain

Resistor

$* \theta_i = -\theta$

$$v = R[I_m \cos(\omega t + \theta_i)$$
$$= RI_m \cos(\omega t + \theta_i)$$
$$\Rightarrow \vec{V} = RI_m e^{j\theta_i} = RI_m \angle \theta_i$$
$$\Rightarrow \vec{V} = R\vec{I}$$

Inductor

$$v = L\frac{di}{dt} = L\frac{d}{dt}\left(I_m \cos(\omega t + \theta_i)\right)$$
$$= -\omega L I_m \sin(\omega t + \theta_i)$$
$$\Rightarrow \quad = -\omega L I_m \cos(\omega t + \theta_i - 90°)$$
$$\vec{V} = -\omega L I_m e^{j\theta_i} e^{-90°}$$
$$\vec{V} = j\omega L I_m e^{j\theta_i} = j\omega L \vec{I}$$

Capacitor

$i = C\frac{dv}{dt} = C\frac{d}{dt}[V_m \cos(\omega t + \theta_v)]$

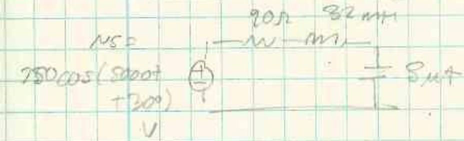
$\vec{I} = j\omega C \vec{V}$

$\Rightarrow \vec{V} = \frac{1}{j\omega C}\vec{I}$

Impedance

$\vec{V} = \vec{Z}\vec{I}$

What is current?



$\vec{Z}_L = j\omega L = j(5000)(32 \cdot 10^{-3}) = j160\Omega$

$\vec{Z}_C = j\frac{-1}{\omega C} = \frac{-j}{(5000)(5)(10^{-6})} = -j40\Omega$

$\vec{V}_s = 750\angle 30°V$

$Z_{tot} = 90 + j160 - j40 = 90 + j120$ $|Z_{tot}| = \sqrt{90^2 + 120^2} = 150$

$\vec{I} = \frac{\vec{V}_s}{Z_{tot}} = \frac{750\angle 30°}{150\angle 53.13°}$ $\theta_{tot} = \tan^{-1}\left(\frac{120}{90}\right) = 53.13°$

$= \boxed{5\angle -23.13° A}$